

**From:** [Moody, Dustin \(Fed\)](#)  
**To:** (b) (6)  
**Subject:** RE: a very large Galois group, so that the number field is very far from having automorphisms.  
**Date:** Thursday, April 26, 2018 2:37:00 PM

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This seems to be true for NTRUprime. I don't see where they claim the attack doesn't work.

**From:** Quynh Dang (b) (6)  
**Sent:** Thursday, April 26, 2018 2:30 PM  
**To:** Moody, Dustin (Fed) <[dustin.moody@nist.gov](mailto:dustin.moody@nist.gov)>  
**Subject:** Re: a very large Galois group, so that the number field is very far from having automorphisms.

Here is the attack: Ciphertext :  $c = rh + m$ .

Number of 1s = number of -1s in r, so  $r(1) = 0$  which implies  $c(1) = r(1)h(1) + m(1) = m(1)$  which reveals information about m. If  $c(1)$  is a huge positive number which means there are way more 1s than -1s which means that in m there are way more 1s than -1s: this gives information about m.

Quynh.

On Thu, Apr 26, 2018 at 2:25 PM, Moody, Dustin (Fed) <[dustin.moody@nist.gov](mailto:dustin.moody@nist.gov)> wrote:

Write out the attack. Explain it to me....

**From:** Quynh Dang (b) (6)  
**Sent:** Thursday, April 26, 2018 2:18 PM  
**To:** Moody, Dustin (Fed) <[dustin.moody@nist.gov](mailto:dustin.moody@nist.gov)>  
**Subject:** Re: a very large Galois group, so that the number field is very far from having automorphisms.

Why not having subfield or subring stops that attack ?

Quynh.

On Thu, Apr 26, 2018 at 2:12 PM, Moody, Dustin (Fed) <[dustin.moody@nist.gov](mailto:dustin.moody@nist.gov)> wrote:

That it doesn't have subrings (i.e. subfields), except the trivial ones.

**From:** Quynh Dang (b) (6)  
**Sent:** Thursday, April 26, 2018 2:11 PM  
**To:** Moody, Dustin (Fed) <[dustin.moody@nist.gov](mailto:dustin.moody@nist.gov)>  
**Subject:** Re: a very large Galois group, so that the number field is very far from having automorphisms.

So, what actually stops the attack in NTRU prime ?

On Thu, Apr 26, 2018 at 2:09 PM, Moody, Dustin (Fed) <[dustin.moody@nist.gov](mailto:dustin.moody@nist.gov)> wrote:

Yes, it works on fields and rings. But it involves a subring....

**From:** Quynh Dang (b) (6)  
**Sent:** Thursday, April 26, 2018 2:03 PM  
**To:** Moody, Dustin (Fed) <[dustin.moody@nist.gov](mailto:dustin.moody@nist.gov)>  
**Subject:** Re: a very large Galois group, so that the number field is very far from having automorphisms.

Can you correct me below Dustin ?

On Thu, Apr 26, 2018 at 9:49 AM, Quynh Dang (b) (6) wrote:

Thank you Dustin.

Below is my understanding of the attack (wrong understanding).

Ciphertext :  $c = rh + m$ . Number of 1s = number of -1s in r, so  $r(1) = 0$  which implies  $c(1) = r(1)h(1) + m(1) = m(1)$

So, my wrong understanding is that the attack works for rings or fields.

Quynh.

On Thu, Apr 26, 2018 at 9:44 AM, Moody, Dustin (Fed) <[dustin.moody@nist.gov](mailto:dustin.moody@nist.gov)> wrote:

Quynh,

I don't understand the statement about having a very large Galois group means the number field is very far from having automorphisms. By definition, the Galois group elements are automorphisms. So a large Galois group would mean a lot of automorphisms. I've read the blog, but I still can't make sense of it.

I think that a field blocks the evaluation at 1 attacks because the attack works with a subring. For a field, the subring is either the entire field or just {1}, which isn't helpful. By the way  $\phi_n(x) = (x^n - 1)/(x - 1) = x^{n-1} + x^{n-2} + \dots + x + 1$ . This will be irreducible if n is prime. The fact that it is irreducible means when we do  $\mathbb{Q}[x] / \phi_n(x)$  we get a field.

Dustin

**From:** Quynh Dang (b) (6)  
**Sent:** Thursday, April 26, 2018 8:59 AM  
**To:** Moody, Dustin (Fed) <[dustin.moody@nist.gov](mailto:dustin.moody@nist.gov)>

**Subject:** a very large Galois group, so that the number field is very far from having automorphisms.

Hi Dustin,

On a Dan's blog article: <https://blog.cr.yp.to/20140213-ideal.html>, he said that " and uses an irreducible polynomial  $x^p-x-1$  with a very large Galois group, so that the number field is very far from having automorphisms. " .

Why is this harder to find automorphisms if the Galois group is large ?

Why  $R/q$  (defined in NTRU prime) (a field instead of a ring) avoids evaluation at  $m(1)$  attack ? The attack seems to work as long as the number of  $-1$  and  $1$  coefficients are known in  $r$  (I think my understanding for the attack is wrong here) because Tanga claims that replacing  $X^N - 1$  in the original NTRU with  $(X^N - 1)/(x - 1)$  to avoid the attack.

If the claim is correct, my impression is that  $(X^N - 1)/(x - 1)$  is irreducible ( I dont know this is true or not). If this is true, why does it being irreducible avoids the attack ?

Thank you!  
Quynh.